The Empirical Econometrics and Quantitative Economics Letters
ISSN 2286 – 7147
Volume 3, Number 1 (March 2014), pp. 25 - 32.

Mathematical proofs of the middle path in Buddhist economics as a pathway to the maximum of happiness

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ABSTRACT

This paper investigates the pathway to the maximum happiness following the middle path in Buddhist Economics. It constructs utility functions and production functions that an individual will be happy when he or she spends time and financial assets for self and public interests. By dynamic optimization to maximize the utility under constraints of constant outputs done for self-interest and public interest over time, the solutions discovers the trade-offs between the time and financial assets spent for self-interest, and also reveals the trade-off between time and the assets donated for public interests as the mean to happiness. Moreover, it shows that an individual cannot live in an extreme way by spending all the time for self-interest while spend all financial assets for other people and vice versa. Therefore, this study mathematically confirms that the middle path in Buddhist Economics is consistent with the optimality in economics, and it is the pathway to the maximum of happiness.

Keywords: Dynamic optimization, simulation modeling, Buddhist economics, microeconomic behavior, utility function

JEL Classification: C61, C63, D01

1. Introduction

In Buddhist Economics, the middle path may be the most important solution to gain the maximized happiness. However, there is no mathematical proof of it. This paper tries to find the mathematical solution on this issue.

This study assumes the perception of utility via two outputs, the one done for self-interest and the other one done for public interest. The production for the output for self-interest composes of two factors of production, time and financial assets allocated for self interest. Similarly, the production of output for public interest uses two factors which are time and financial assets arranged for public interest.

The maximization over time uses Hamiltonian function whose objective function is the utility function and the constraints are the constant outputs done for self and public interests over time. The dynamic optimization will yield the optimal level of all factors of production. These solutions will guide an individual whether he or she should follow the middle path or not.

2. Literature review

The closest paper on mathematical modeling in Buddhist Economics is of Suriya (2011). In that paper, the author proposes some formations of utility function that diminishes along with the amount of consumption. That is there must be a maximum of utility such that an individual should not consumer more than that optimal level. This may be concerned as the middle path in Buddhist Economics. However, the paper does not extend its scope to find the optimality in production which is the gap that this study will fill.

3. Settings of the model

3.1 The utility function

As described in section 1, the model sets the utility function as follows:

$$U = C[\alpha m^{-\psi} + \beta w^{-\psi}]^{-\frac{1}{\psi}}$$

where U = utility of an individual,

m = output that an individual produces for self-interest,

w = output that an individual produces for other people,

C, α, β, ψ = parameters,

such that $\psi = \frac{1-\pi}{\pi}$ when π is the elasticity of substitution between m and w.

3.2 The production functions

It also sets the production functions as follows:

$$m = A[\lambda c^{-\rho} + \delta k^{-\rho}]^{-\frac{1}{\rho}}$$

where m = output that an individual does for self-interest,

c = financial assets kept for self-interest,

k = time spent for self-interest,

A, λ, δ, ρ = parameters,

such that $\rho = \dfrac{1-\varepsilon}{\varepsilon}$ when ε is the elasticity of substitution between c and k.

$$w = B[\gamma g^{-\rho} + \theta s^{-\rho}]^{-\frac{1}{\rho}}$$

where w = output that an individual does for other people,

g = financial assets donated for public interest,

s = time spent for other people,

B, γ, θ, ρ = parameters,

and $\rho = \dfrac{1-\varepsilon}{\varepsilon}$ when ε is also the elasticity of substitution between g and s.

4. Dynamic optimization

The Hamiltonian function can be written as follows:

$$H = C[\alpha m^{-\psi} + \beta w^{-\psi}]^{-\frac{1}{\psi}} + \eta(\dot{m}) + \mu(\dot{w})$$

where

$$\dot{m} = \frac{dm}{dt} = A(\lambda c^{-\rho} + \delta k^{-\rho})^{-\frac{1}{\rho}-1} \cdot (\lambda c^{-\rho-1} \cdot \frac{dc}{dt} + \delta k^{-\rho-1} \cdot \frac{dk}{dt})$$

$$\dot{w} = \frac{dw}{dt} = B(\gamma g^{-\rho} + \theta s^{-\rho})^{-\frac{1}{\rho}-1} \cdot (\gamma g^{-\rho-1} \cdot \frac{dg}{dt} + \theta s^{-\rho-1} \cdot \frac{ds}{dt})$$

Find the first derivative of the Hamiltonian function subjected to m, w and Hamiltonian's multipliers to maximize the utility under constraints $\dot{m} = 0$ and $\dot{w} = 0$.

$$\frac{\partial H}{\partial m} = C \cdot [\alpha m^{-\phi} + \beta w^{-\phi}]^{-\frac{1}{\phi}-1} \cdot (\alpha m^{-\phi-1}) = 0$$

$$\frac{\partial H}{\partial w} = C \cdot [\alpha m^{-\phi} + \beta w^{-\phi}]^{-\frac{1}{\phi}-1} \cdot (\beta w^{-\phi-1}) = 0$$

$$\frac{\partial H}{\partial \eta} = \mathrm{A}(\lambda c^{-\rho} + \delta k^{-\rho})^{-\frac{1}{\rho}-1} \cdot (\lambda c^{-\rho-1}\dot{c} + \delta k^{-\rho-1}\dot{k}) = 0$$

$$\frac{\partial H}{\partial \mu} = \mathrm{B}(\gamma g^{-\rho} + \theta s^{-\rho})^{-\frac{1}{\rho}-1} \cdot (\gamma g^{-\rho-1}\dot{g} + \theta s^{-\rho-1}\dot{s}) = 0$$

The solutions are as follows:

$$c = -(\frac{\lambda}{\delta})^{1/\rho} \cdot k$$

$$g = -(\frac{\gamma}{\theta})^{1/\rho} \cdot s$$

However, c and k are in the function m while g and s are in function w.

$$m = A[\lambda c^{-\rho} + \delta k^{-\rho}]^{-\frac{1}{\rho}}$$

$$w = B[\gamma g^{-\rho} + \theta s^{-\rho}]^{-\frac{1}{\rho}}$$

Find the shape of function c in terms of k.

$$c = -(\frac{\lambda}{\delta})^{1/\rho} \cdot k$$

$$\frac{\partial c}{\partial k} = -\frac{1}{\rho}(\frac{\lambda}{\delta})^{1/\rho - 1}$$

It is clear that the shape of the function is linear with the trade-off between c and k.

Find the shape of function g in terms of s.

$$g = -(\frac{\gamma}{\theta})^{1/\rho} \cdot s$$

$$\frac{\partial g}{\partial s} = -\frac{1}{\rho}(\frac{\gamma}{\theta})^{1/\rho - 1}$$

This is again that the shape of the function is linear exhibiting the trade-off between g and s.

Find the shape of the explicit function m in the c-k space.

$$m = A[\lambda c^{-\rho} + \delta k^{-\rho}]^{-\frac{1}{\rho}}$$

$$dm = A[\lambda c^{-\rho} + \delta k^{-\rho}]^{-1/\rho - 1} \cdot [\lambda c^{-\rho-1} \cdot dc + \delta k^{-\rho-1} \cdot dk]$$

On the same isoquant function, $dm = 0$

$$\lambda c^{-\rho-1} \cdot dc + \delta k^{-\rho-1} \cdot dk = 0$$

$$\lambda c^{-\rho-1} \cdot dc = -\delta k^{-\rho-1} \cdot dk$$

$$\frac{dc}{dk} = \frac{-\delta k^{-\rho-1}}{\lambda c^{-\rho-1}}$$

Due to the positive values of all parameters with the positive values of c and k, the slope of the function is negative.

Find the second derivative,

$$\frac{d^2 c}{dk^2} = (-\rho - 1)\frac{-\delta k^{-\rho-2}}{\lambda c^{-\rho-1}} = (\rho + 1)\frac{\delta k^{-\rho-2}}{\lambda c^{-\rho-1}}$$

Therefore, the second derivative is positive, then the function is convex.

Find the shape of the explicit function w in the g-s space.

$$w = B[\gamma g^{-\rho} + \theta s^{-\rho}]^{-\frac{1}{\rho}}$$

$$dw = B[\gamma g^{-\rho} + \theta s^{-\rho}]^{-1/\rho - 1} \cdot [\gamma g^{-\rho-1} \cdot dg + \theta s^{-\rho-1} \cdot ds]$$

On the same isoquant function, $dw = 0$

$$\gamma g^{-\rho-1} \cdot dg + \theta s^{-\rho-1} \cdot ds = 0$$

$$\gamma g^{-\rho-1} \cdot dg = -\theta s^{-\rho-1} \cdot ds$$

$$\frac{dg}{ds} = \frac{-\theta s^{-\rho-1}}{\gamma g^{-\rho-1}}$$

Find the second derivative,

$$\frac{d^2 g}{ds^2} = (-\rho-1)\frac{-\theta s^{-\rho-1}}{\gamma g^{-\rho-1}} = (\rho+1)\frac{\theta s^{-\rho-1}}{\gamma g^{-\rho-1}}$$

The second derivative is also positive showing that the function is convex too.

5. Solutions

5.1 Solutions for c and k

To find the solution of c and k, the slope of both functions must be equal.

$$\frac{\partial c}{\partial k} = -\frac{1}{\rho}(\frac{\lambda}{\delta})^{1/\rho-1}$$

$$\frac{dc}{dk} = \frac{-\delta k^{-\rho-1}}{\lambda c^{-\rho-1}}$$

That is

$$\frac{\lambda c^{-\rho-1}}{\rho}(\frac{\lambda}{\delta})^{1/\rho-1} = \delta k^{-\rho-1}$$

$$\left\{\left(\frac{\lambda}{\delta\rho}\right)^{\frac{-1}{(\rho+1)}}(\frac{\lambda}{\delta})^{\frac{\rho-1}{\rho(\rho+1)}}\right\}c = k$$

$$k = c\left\{\left(\frac{\delta}{\lambda}\right)^{\frac{1}{\rho(\rho+1)}}\rho^{\frac{1}{\rho+1}}\right\}$$

$$k = c\left\{\left(\frac{\delta}{\lambda}\right)^{\frac{1}{\rho}}\rho\right\}^{\frac{1}{\rho+1}}$$

Therefore, k is a positive number. It cannot be zero. The solution is not the corner solution. In case that all parameters are one, then k equals to c. In case that all parameters are less than one, e.g. 0.1, k is less than c. In contrast, when all parameters are greater than one, e.g. 1.9, k is greater than c.

5.2 Solutions for g and s

To find the solution of g and s, the slope of both functions must be equal.

$$\frac{\partial g}{\partial s} = -\frac{1}{\rho}\left(\frac{\gamma}{\theta}\right)^{1/\rho - 1}$$

$$\frac{dg}{ds} = \frac{-\theta s^{-\rho-1}}{\gamma g^{-\rho-1}}$$

The solutions follow the same fashion of the previous section.

$$s = g\left\{\left(\frac{\theta}{\gamma}\right)^{\frac{1}{\rho}} \rho\right\}^{\frac{1}{\rho+1}}$$

Clearly, s cannot be zero. It's value depends on the sizes of parameters. In case that all parameters are less than one, s will be less than g. In the opposite side, when all parameters are larger than unity, then s is greater than g.

6. Conclusions

This paper explores Buddhist Economics in a mathematical way. It constructs utility function as a combination of satisfaction between outputs done for both self and public interests. It also sets the productions of both outputs as products of two factors of production, i.e. time and financial assets spent for self or public interests. It aims at finding the solutions by dynamic optimization which maximizes the utility under constraints of constant outputs over time. It would like to answer whether an individual should behave following the middle path according to Buddhist Economics or not.

The mathematical solutions discover that there are trade-off between time and financial assets that an individual can keep for self-interest. The trade-off also appears between the time and financial assets allocated for other people. It means that when an individual spend time for self-interest, he or she must donate financial assets to other people in order to achieve the maximized utility. On the other hand, if an individual works too hard for public interest, he or she must be compensated heavily by financial assets too.

The solutions also show that the middle path is the optimality of the choice between time and financial assets spent for both self and public interests. Nothing can be extreme such that an individual spend the whole time for himself or herself while spend all the financial assets to other people. Such the extreme cannot yield maximized utility. The optimality is to keep some time and some portions of financial assets for self interest while donate some time and also some portions of the assets to public interests too. These results confirm the middle path as a pathway to the maximum of happiness.

REFERENCES

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