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Modeling risk of foreign exchange portfolio based on GARCH-EVT-copula and filtered historical simulation approaches

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ABSTRACT

This study shows how to model the financial market risk of a foreign exchange portfolio using a GJR-GARCH-EVT-Copula approach versus a filtered historical simulation (FHS) technique. The first method extracts the filtered residuals from each return series with an asymmetric GARCH model, and then constructs the sample marginal cumulative distribution function (CDF) of each asset using a Gaussian kernel estimate for the interior and a generalized Pareto distribution (GPD) estimate for the upper and lower tails. A Student's *t* copula is then fitted to the data and used to induce correlation between the simulated residuals of each asset; whereas, FHS combines a relatively sophisticated model-based treatment of volatility (GARCH) with a nonparametric specification of the probability distribution of assets returns. One of the appealing features of FHS is its ability to generate relatively large deviations (losses and gains) not found in the original portfolio return series. FHS retains the nonparametric nature of historical simulation by bootstrapping (sampling with replacement) from the standardized residuals. These bootstrapped standardized residuals are then used to generate time paths of future asset returns. The simulation assesses the Value-at-Risk (VaR) of the hypothetical global equity portfolio over one-month risk horizon. Finally, we compare simulated VaR by a Monte Carlo simulation technique using a Student's t copula and EVT against simulated VaR by a FHS. Empirical results show that the GJR-GARCH-EVT-Copula model performs better than the filtered historical simulation.

Keywords: Copula Function, Exchange Rate, Extreme Value Theory, Filtered Historical Simulation, GARCH

JEL Classification: C15, C22, F31

1. Introduction

During the last few decades the currency market has been a hotly debated issue in the academic research literature. Particularly the global environment of the foreign exchange market, it is essential to study some of the important historical events relating to currencies and currency exchange. As is well known, today the currency market is the most volatile and liquid in all financial market in the world.

Value-at-Risk (VaR) is one of the most important and widely used statistics that measures the potential of economic losses. In mathematical terms, VaR corresponds to a percentile of portfolio P&L, and can be expressed as potential loss from the current value of the portfolio, or as the loss from the expected value at the horizon.

In general, VaR is calculated either based on Historical Simulation (HS) approach, which imposes virtually no structure on the distribution of returns except stationary, or using Monte Carlo simulation (MCS) approach which assumes parametric models for variance and subsequently large sample of random numbers are drawn from this specific distribution to calculate the desired risk measure. Filtered Historical Simulation (FHS) approach attempts to combine the best of the model-based with the best of the model-free approaches in a very intuitive fashion.

The purpose of this study is to estimate the portfolio VaR of returns in the five markets using a GJR-GARCH-EVT-Copula approach, and then, compare it with a filtered historical simulation (FHS) technique.

This paper is organized as follows. Section 2 presents the literature review. Section 3 introduces the methodology. Section 4 describes the data used in the study. Section 5 discusses the empirical results. Section 6 concludes.

2. Literature review

There is hardly any study which estimated VaR following Filtered Historical Simulation approach using GARCH model with suitable mean specification, in the context of Indian capital market. Pattanaik and Chatterjee (2000) used ARCH/GARCH models to model the volatility in Indian financial markets.

Engle (1982) introduced the concept of Autoregressive Conditional Heteroskedasticity (ARCH) which became a very powerful tool in the modelling of high frequency financial data. ARCH models allow the conditional variances to change through time as functions of past errors. Bollerslev (1986) made significant improvement on ARCH and introduced the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process. Nelson (1991) was introduced the exponential GARCH (EGARCH), where different re-specification of variance equation was studied.

Angelis, et al., (2003) evaluated the performance of an extensive family of ARCH models daily VaR of perfectly diversified portfolios in five stock indices, using a number of distribution assumptions and sample sizes.

The general financial time series are leptokurtic, with heavy-tails which make VaR being underestimated for i.i.d Gaussian distribution. Ho at al., (2000); McNeil and Frey, (2000); Gencay et al., (2003) tend to adopt the extreme value theory (EVT) to solve the problem. Contrary to VaR approaches, EVT is used to model the behavior of maxima or minima in a series (the tail of the distribution). Besides, researchers usually adopted

MLE to estimate the parameters of EVT, but under limited samples MLE causes estimation bias easily.

On the time series characteristics, integrating EVT with time series model evolves into conditional version of EVT (CEVT, or dynamic EVT). McNeil and Frey, (2000); indicates that CEVT employing time series model filters the autocorrelations and heteroskedasticity in finance data. Consequently, the accuracy of VaR estimation is significantly enhanced.

The application of the copula function was first introduced by Li, (2000). McNeil et al., (2005) described that the copula approach provides a way of isolating the description of the dependence structure. It is of course only one way of treating dependence in multivariate risk models and is perhaps most natural in a static distributional context rather a dynamic time series one.

3. Methodology

In this section, we briefly describe the extreme value theory and the copula functions principles useful to construct our multivariate distribution.

A. Extreme Value Theory (EVT)

Extreme Value Theory is used to model the risk of extreme, rare events, etc.

There are two common approaches to POT. One is the semi-parametric models around the Hill estimator (Hill, 1975) and its relatives. The other is the fully parametric models based on the Generalized Pareto Distribution (GPD).

By the theorem of Pickands-Balkema-de Haan, (See Pickands, (1975) and Balkema and de Haan (1974)) stating that the generalized Pareto distribution given by:

$$G_{\xi,\beta}(x) = \begin{cases} 1-\left(1+\xi\dfrac{x}{\beta}\right)^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1-\exp\left(\dfrac{x}{\beta}\right), \xi = 0 \end{cases} \quad (1)$$

Where: $\beta > 0$ is the scale parameter, and $x \geq 0$ when $\xi \geq 0$ is the shape parameter and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$, appears as the limit distribution of scaled excesses over high thresholds. More precisely, it can be shown (see Embrechts, et al., (1997), Theorem 3.4.13(b)) that for a large class random variable *X* there exists a function $\beta(\cdot)$ such that

$$\lim_{u \to x_F} \sup_{0 \leq x < x_F - u} \left| F_u(u) - G_{\xi,\beta(u)}(u) \right| = 0 \quad (2)$$

Where: $x_F \leq \infty$ denotes the right endpoint of the distribution function of *X*.

B. Copula Function

Copula functions are a useful tool to construct and simulate multivariate distributions.

Definition: An *n*-dimensional copula is a multivariate cumulative distribution function, C with uniform distributed margins on $[0,1]$ $(U(0,1))$ and the following properties:

1. $C:[0,1]^n \to [0,1]$;
2. C is grounded and *n*-increasing;
3. C has margins C_i which satisfy $C_i(u) = C(1,\ldots,1,u,1,\ldots,1) = u$ for all $u \in [0,1]$.

Sklar's Theorem (see, Sklar, 1959) is the most important theorem regarding to copula functions since it is used in many practical applications (see, Li, (2000)).

4. Data

The data for this study are extracted from Quandl website. The dataset used consists of 2278 observations of daily closing values of the exchange rates for the five countries, following daily currency indices, from 22 July 2005 to 16 April 2014: Yuan/Dollar (CNY/USD), Euro/Dollar (EUR/USD), Yen/Dollar (JPY/USD), CFA Franc/Dollar (XAF/USD), and Rand/Dollar (ZAR/USD).

5. Results and Discussions

GJR-GARCH-EVT-Copula approach

These exchange rate indices are then transformed into daily returns are shown in Figure 1. It's obvious that they are highly correlated.

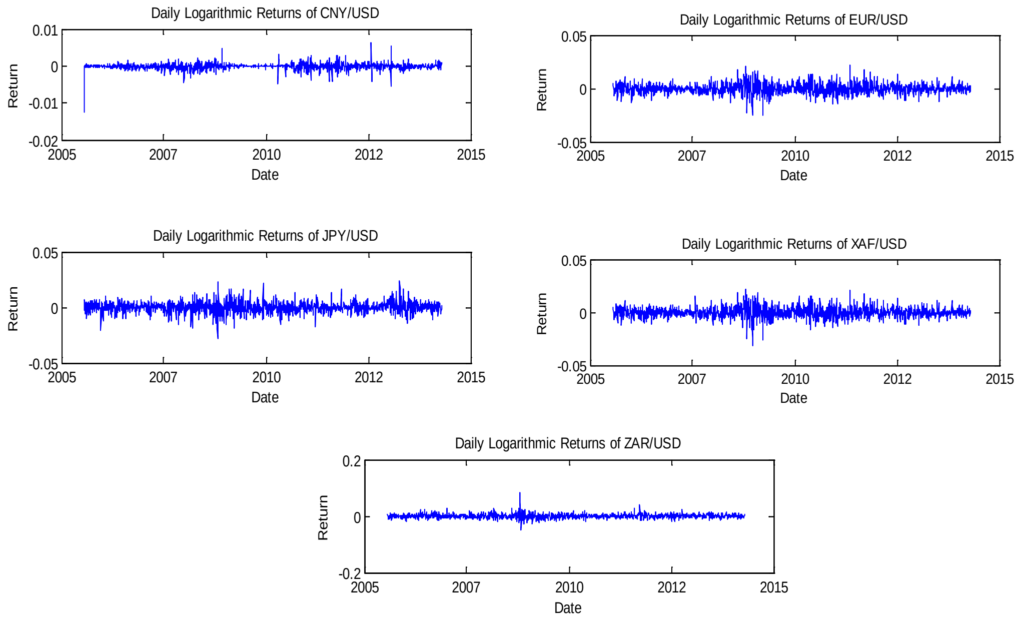

Figure 1. Daily logarithmic returns of the five countries exchange rate

Before using EVT to model the tails of the distribution of an individual index, we must ensure that the data is approximately independent and identically distributed (i.i.d).

However, most financial return series exhibit some degree of autocorrelation and, more importantly, heteroskedasticity. Below the different figures show the sample ACF of returns and sample ACF of squared returns for a particular country.

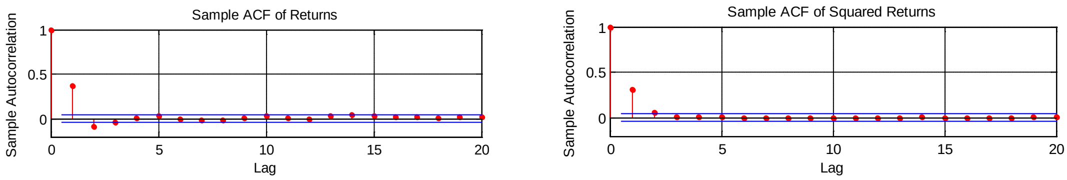

Figure 2. Sample ACF of returns and sample ACF of squared returns of CNY/USD

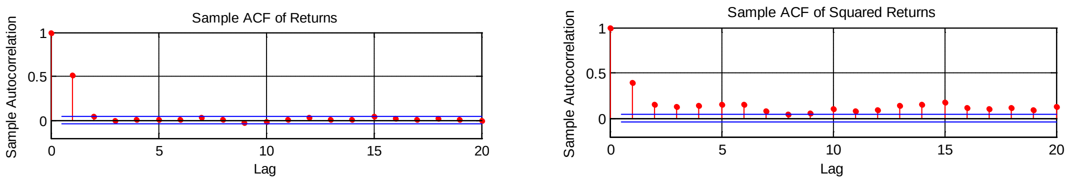

Figure 3. Sample ACF of returns and sample ACF of squared returns of EUR/USD

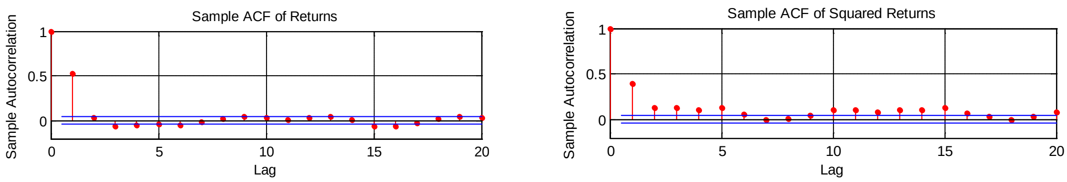

Figure 4. Sample ACF of returns and sample ACF of squared returns of JPY/USD

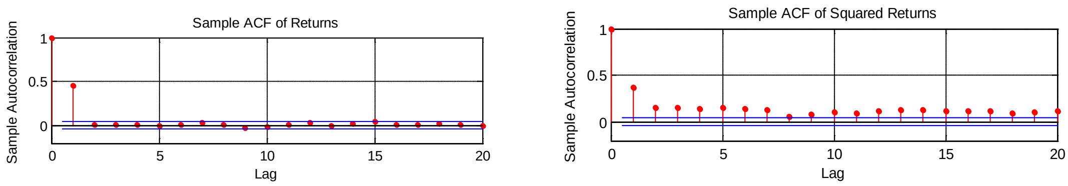

Figure 5. Sample ACF of returns and sample ACF of squared returns of XAF/USD

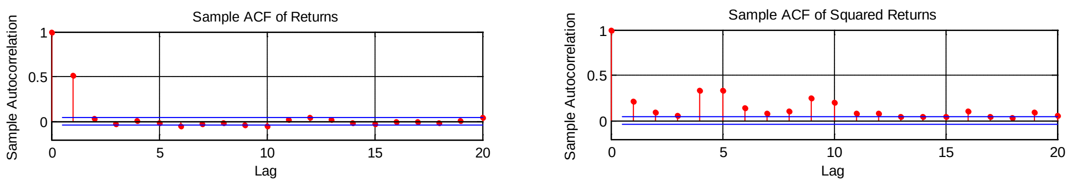

Figure 6. Sample ACF of returns and sample ACF of squared returns of ZAR/USD

The sample ACF of the squared returns illustrates the degree of persistence in variance, and implies that we need a GARCH model to condition the data for the subsequent tail estimation process. Consequently, we use a GARCH model to filter out serial dependence in the data. Even though returns are not independent from one day to the next, the AR(1)-GJR-GARCH(1, 1) model produces a series of independent and identically distributed observations that let us to more closely satisfy the requirements of EVT. We fit a AR(1)-GJR-GARCH(1, 1) model as follows to each index:

$$r_t = c + \theta r_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N\left(0, \sigma_t\right) \quad (3)$$

$$\sigma^2 = k + \alpha\sigma_{t-1}^2 + \phi\varepsilon_{t-1}^2 + \psi\varepsilon_{t-1}^2 I_{t-1} \quad (4)$$

Where: $I_{t-1} = \begin{cases} 0 & if \quad \varepsilon_{t-1} \geq 0 \\ 1 & if \quad \varepsilon_{t-1} < 0 \end{cases}$ and r_t is the index return, and σ_t the volatility.

After that, we compared the model residuals and the corresponding conditional standard deviations filtered form the raw returns.

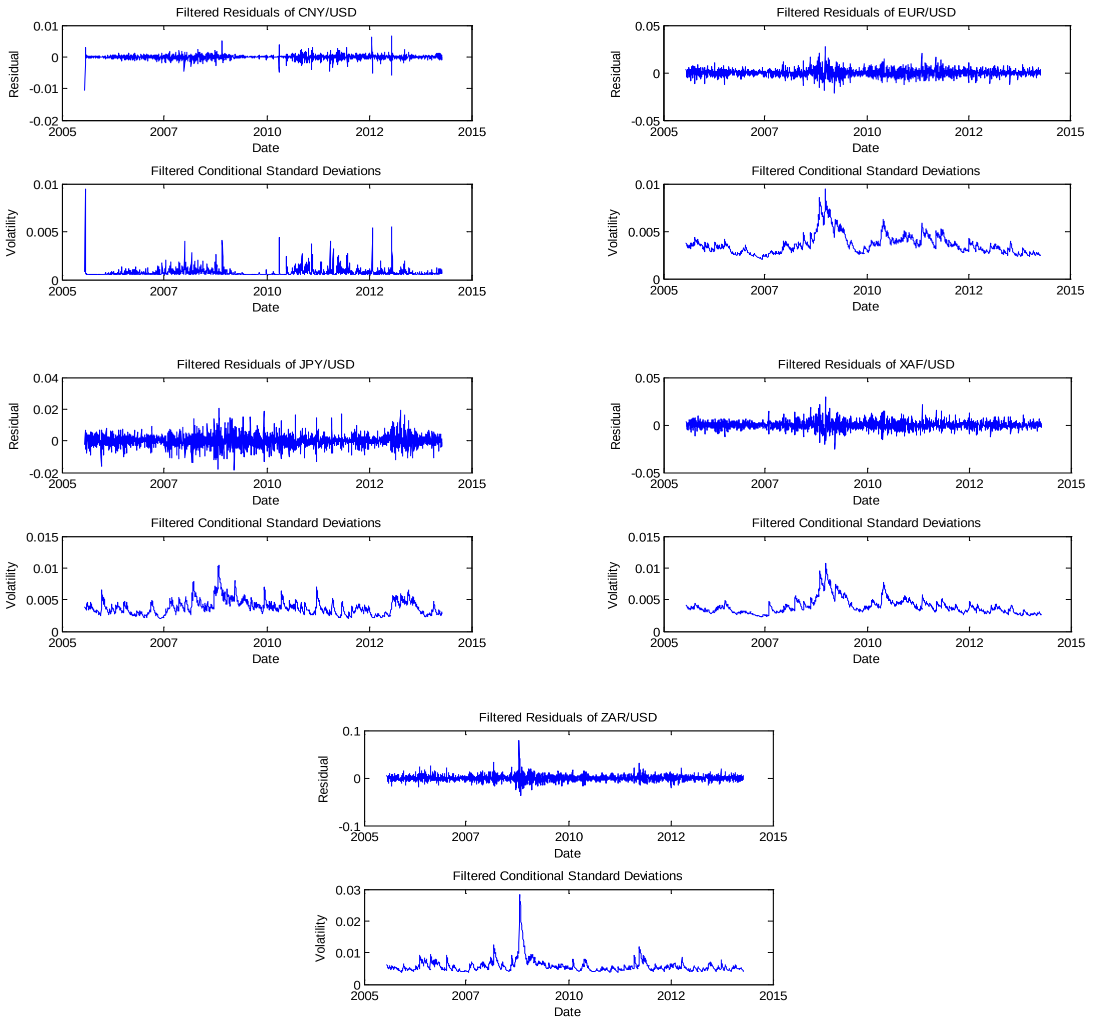

Figure 7. Filtered residuals and filtered conditional standard deviations for a particular country

Each lower graph of figure 7 clearly illustrates the variation in volatility present in the filtered residuals. Subsequently, we standardize the residuals by the corresponding conditional standard deviation. These standardized residuals represent the underlying zero-mean unit-variance, i.i.d series upon which the EVT estimation of the sample CDF tail is based.

Examine the ACF of the standardized residuals and squared standardized residuals.

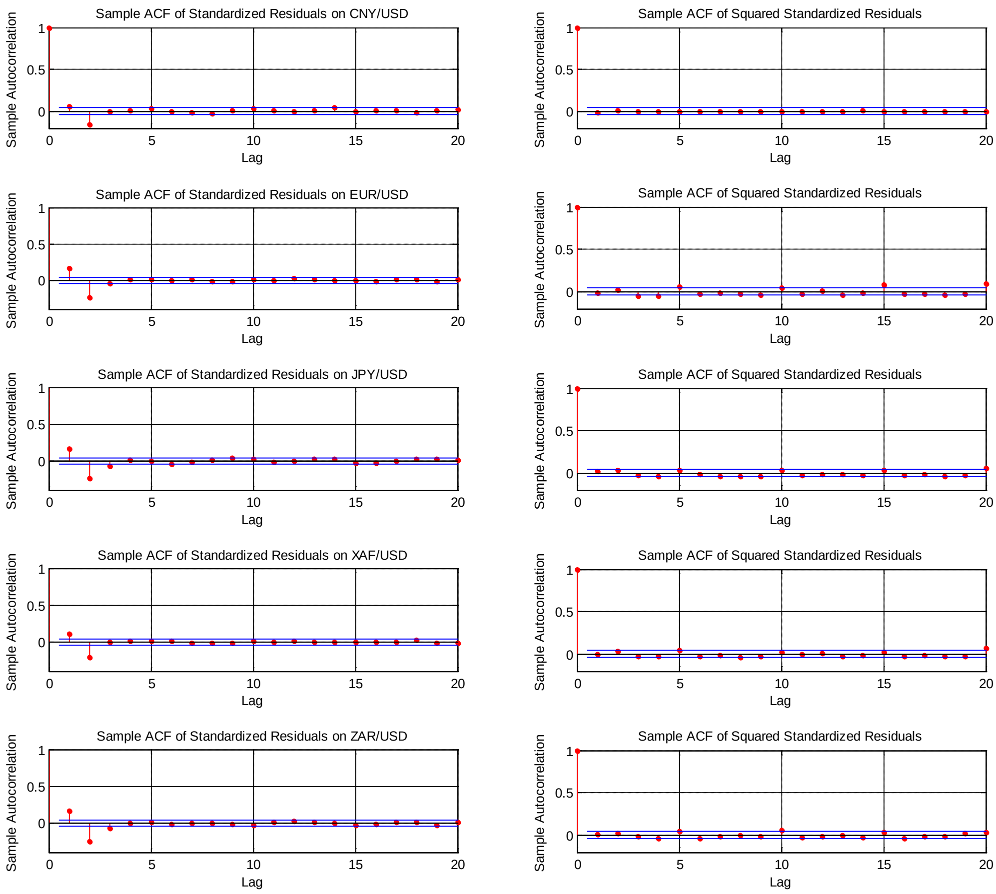

Figure 8. Sample ACF of the standardized residuals and squared standardized residuals for each index

We fit a probability distribution to model the daily movements of each index, after filtering the data. We do not assume that the data comes from a normal distribution or from any other simple parametric distribution.

A kernel density estimate works well for the interior of the distribution where most of the data is found, but it performs poorly when applied to upper and lower tails. In risk management, it is essential to accurately characterize the tails of the distribution, even though the observed data in the tails is sparse. The generalized Pareto distribution (GPD) is often used for this purpose. Figure 9 shows the empirical cumulative distribution function (CDF) with the kernel density estimate for the interior and the GPD estimate for the upper and lower tails for a particular country.

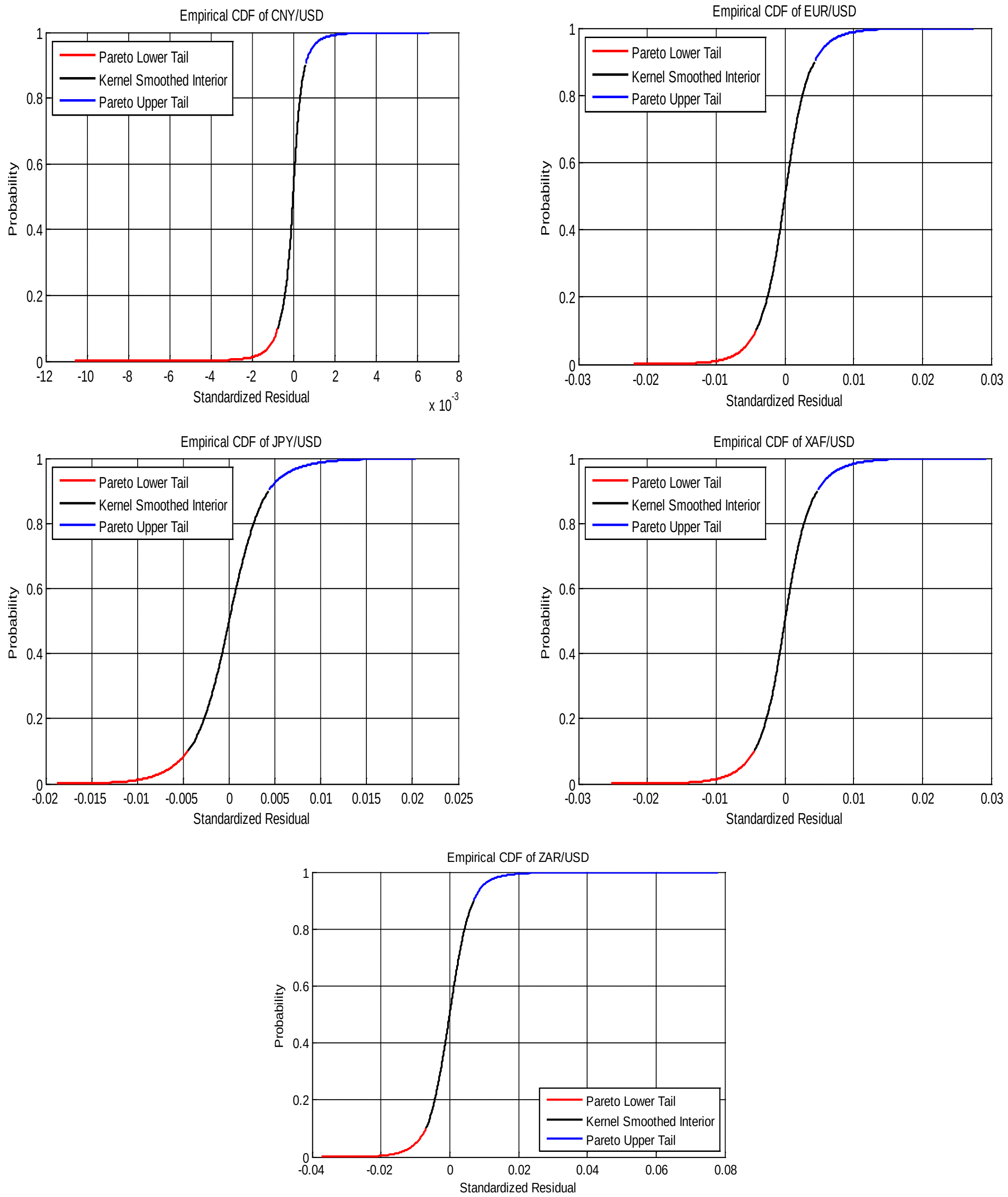

Figure 9. Empirical CDF for a particular country

Having estimated the three distinct regions of the composite semi-parametric empirical CDF, graphically concatenate and display the results. We note that the lower and upper tail regions, displayed in red and blue, are suitable for extrapolation, while the kernel-smoothed interior, in black, is suitable for interpolation.

Although the previous graph illustrates the composite CDF, it is instructive to examine the GDP fit in more detail. The CDF of a GP distribution is parameterized as:

$$F(y) = 1 - (1 + \xi y/\beta)^{-1/\xi}, y \geq 0, \beta > 0, \xi > -0.5 \quad (5)$$

for exceedances (y), tail index parameter ξ and scale parameter β.

Plotting the empirical CDF of the upper tail exceedances of the residuals along with the CDF fitted by the GPD.

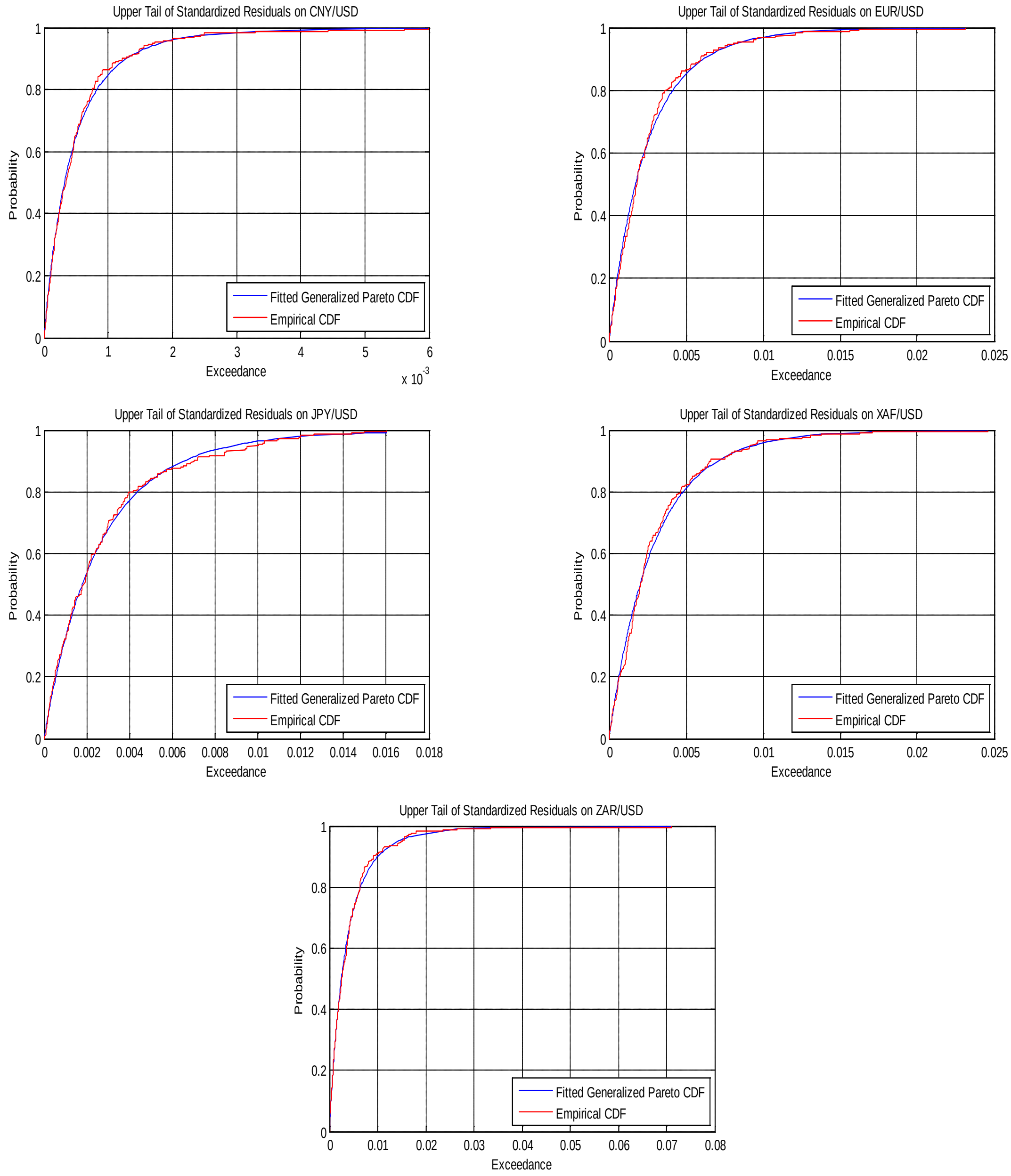

Figure 10. Assessing the GPD fit for a particular country

Figure 10 shows that the empirically generated CDF curve for a particular country matches quite well with the fitted GPD results.

Although only 10% of the standardized residuals is used, the fitted distribution closely follows the exceedances data, so the GPD model seems to be a good choice.

We will have five separate univariate models, one for each of the five indices, describing the distribution of daily gains and losses. But we still need to tie these separate models together, and that is what the copula model does.

Since a copula is a multivariate probability distribution whose individual variables are uniformly distributed, we can now use the univariate distributions that we just derived to transform the individual data of each index to the uniform scale, the form required to fit a copula.

We calibrate the *t* copula by estimating its scalar degrees-of-freedom parameter its linear correlation matrix by maximum likelihood. We found that the degree-of-freedom $\nu = 7.2170$ and the *R* the correlation matrix.

Table 1. Correlation matrix

	CNY/USD	EUR/USD	JPY/USD	XAF/USD	ZAR/USD
CNY/USD	1				
EUR/USD	0.3525	1			
JPY/USD	0.1705	0.2664	1		
XAF/USD	0.3232	0.8655	0.2275	1	
ZAR/USD	0.2406	0.4937	-0.0241	0.4315	1

Filtered Historical Simulation (FHS)

We plot the daily closing value of the hypothetical portfolio along with the corresponding return series for comparison.

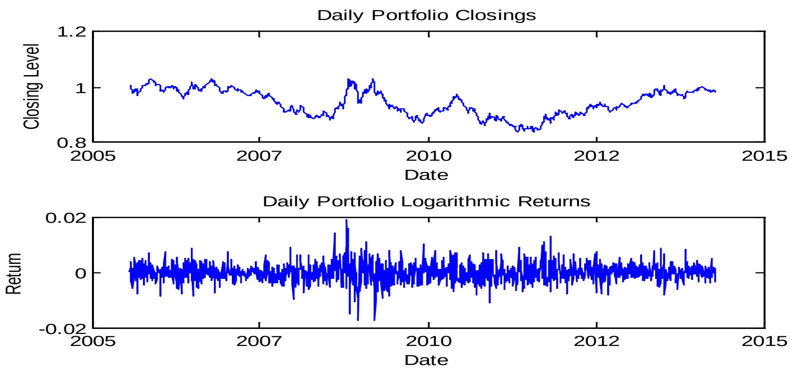

Figure 11. Plot the Daily Portfolio and Daily Portfolio Logarithmic Returns

The bootstrapped FHS method requires the observations to be approximately independent and identically distributed. However, most financial return series exhibit some degree of autocorrelation and more, importantly, heteroskedasticity.

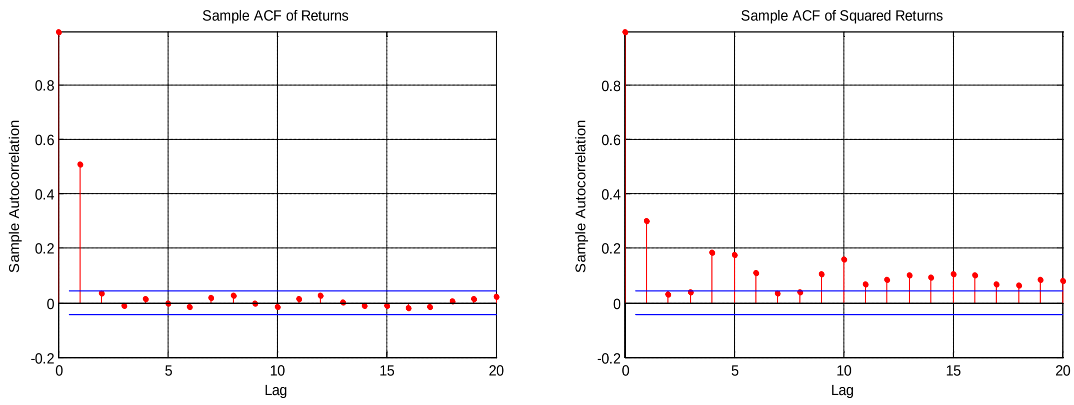

Figure 12. Sample ACF and Sample ACF of Squared of the Portfolio Returns

The sample autocorrelation function (ACF) of the portfolio returns some mild serial correlation. However, the sample ACF of the squared returns illustrates the degree of persistence in variance, and implies that we need a GARCH model may significantly condition the data used in the subsequent bootstrapping method.

To produce a series of i.i.d observations, fit a AR(1)+EGARCH(1,1) model as follows:

$$r_t = c + \theta r_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_t) \quad (6)$$

and an symmetric exponential GARCH (EGARCH) model to the conditional variance

$$\log\left[\sigma_t^2\right] = k + \alpha \log\left[\sigma_{t-1}^2\right] + \phi\left(\left|z_{t-1}\right| - E\left[\left|z_{t-1}\right|\right]\right) + \psi z_{t-1} \quad (7)$$

AR(1) model compensates for autocorrelation, while the EGARCH model compensates for heteroskedasticity.

Table 2. Results of ARIMA (1, 0, 0) Model

ARIMA(1,0,0) Model			
Conditional Probability Distribution: *t*			
Standard *t*			
Parameter	Value	Error	Statistic
Constant	-2.64442e-05	4.42526e-05	-0.597575
AR{1}	0.494797	0.0176657	28.0088
DoF	5.94203	0.744046	7.9861

Table 3. Results of EGARCH (1, 1) Conditional Variance Model

EGARCH(1,1) Conditional Variance Model			
Conditional Probability Distribution: *t*			
Standard *t*			
Parameter	Value	Error	Statistic
Constant	-0.0876156	0.0424002	-2.0664
GARH{1}	0.992746	0.00351942	282.076
ARCH{1}	0.087677	0.0165408	5.30066
Leverage{1}	0.0125096	0.0115071	1.08712
DoF	5.94203	0.744046	7.9861

The estimation shows the six estimated parameters and their corresponding standard errors (the AR(1) conditional mean model has two parameters, and the EGARCH(1,1) conditional variance model has four parameters).

The fitted model is:

$$r_t = -2.64\times10^{-5} + 0.49 r_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N(0,\sigma_t)$$

$$\log\left[\sigma_t^2\right] = -0.08 + 0.99\log\left[\sigma_{t-1}^2\right] + 0.08\left(\left|z_{t-1}\right| - E\left[\left|z_{t-1}\right|\right]\right) + 0.01 z_{t-1}$$

(*t* statistic of AR(1) in ARCH (1,0,0) model is greater than two, meaning than this parameter is statistically significant, while for GARCH and ARCH in EGARCH (1,1) model).

Let's compare the model residuals and the corresponding conditional standard deviations filtered from the raw returns.

The lower graph clearly illustrates the variation in volatility (heteroskedasticity) present in the filtered residuals.

The i.i.d property is important for bootstrapping, and allows the sampling procedure to safely avoid the pitfalls of sampling from a population in which successive observations are serially dependent.

We examine the ACF of the standardized residuals and squared standardized residuals.

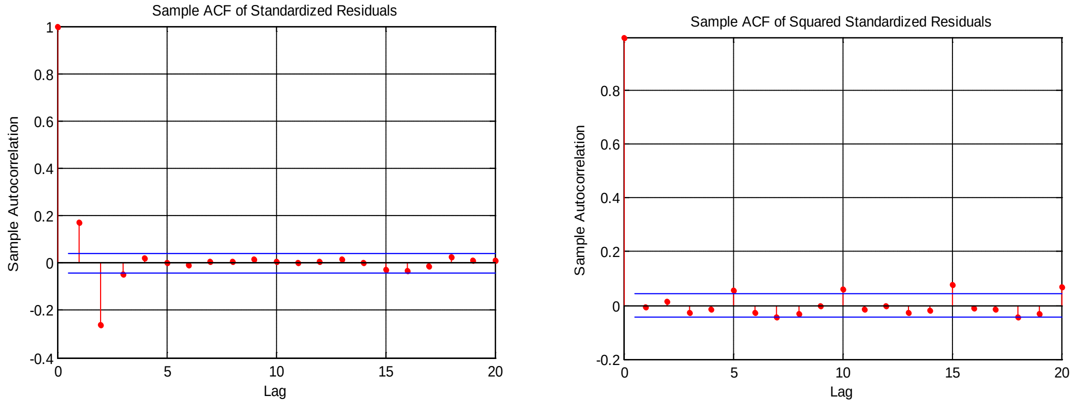

Figure 13. Sample ACF of Standardized Residuals and Sample ACF of Squared Standardized Residuals

Comparing the ACF of the standardized residuals to the corresponding ACF of the raw returns reveals that the standardized residuals are now approximately i.i.d., thereby far more amenable to subsequent bootstrapping.

Computing VaR using a GJR-GARCH-EVT-Copula approach:

We transform the individual standardized residuals of AR (1)-GJR-GARCH (1, 1) models to uniform variates by the semi-parametric empirical CDF, and then fit the *t* copula to the transformed data. The estimated optimal degree of freedom ν of the t copula is 7.2170. This research also adopts t copula with $\nu = 10, 15, 20$ for comparison. Subsequently, this study simulates jointly dependent currency index returns by reversing the above steps.

We simulate 2000 independent random trials of dependent standardized index residuals over a one month horizon of 22 trading days.

Then, using the simulated standardized residuals as the i.i.d., input noise process, reintroduce the autocorrelation and heteroskedasticity of GJR-GARCH model observed in the original index returns. Finally, given the simulated returns of each index, we form a 1/5 equally weighted index portfolio composed of the individual indices, and calculate the VaR at 99% confidence levels, over the one month risk horizon. The estimated 90%, 95%, and 99% VaRs for *t*(7.2170).

Computing VaR using a Filtered Historical Simulation Technique:

We use the bootstrapping procedure produces i.i.d. standardized residuals that consistent with those obtained from AR(1) + EGARCH(1,1) filtering process above.

Using the bootstrapping standardized residuals as the i.i.d input noise process, we reintroduce the autocorrelation and heteroskedasticity observed in the original portfolio return series.

Table 3: Value-at-Risk for the different models

Models	CEVT +*t*(7.2170) copula	CEVT +*t*(10) copula	CEVT +*t*(15) copula	CEVT +*t*(20) copula	Filtered Historical simulation
Max Loss	13.4523%	5.9228%	5.2088%	5.3413%	5.2775%
Max Gain	7.5487%	8.8046%	12.4941%	8.2689%	6.3245%
90% VaR	-2.2019%	-2.1233%	-2.1825%	-2.1281%	-1.6846%
95% VaR	-2.8008%	-2.7897%	-2.7604%	-2.8217%	-2.2489%
99% VaR	-4.1790%	-4.0115%	-4.1493%	-4.0747%	-3.1200%

The table 3 clearly shows that the CEVT-Copula based approach performs best. The filtered historical simulation overestimate the portfolio VaR.

6. Conclusions

This paper estimates the portfolio VaR of return in the five markets based on two composite methods. The first model based on GJR-GARCH-EVT-Copula. It is a two-step approach to simulate and model the dependent stock returns being consistent with historical performance. The copula is to isolate the dependence structure of the portfolio from the description of the individual indices. It is a compelling alternative to the traditional assumption of jointly-normal portfolio returns.

In the second model, filtered historical simulation (FHS) technique is an alternative to traditional historical simulation and Monte Carlo simulation approaches. The mean equation is captured by ARMA model while the volatility is modeled by GARCH model. Finally, the VaR is simulated through FHS.

As the results, the GJR-GARCH-EVT-Copula model performs better that the filtered historical simulation.

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